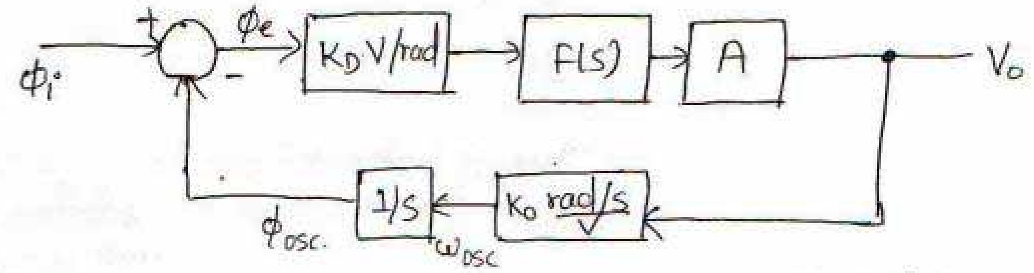
Transfer Function of Phase Locked Loop (PLL)

Under locked conditions, a linear relationship exists betn the output voltage of phase detector and the phase difference betn the VCO and the incoming signal. This fact allows the loop to be analyzed using standard linear feedback concepts when in the locked conditions. A block diagram representation of the system in this mode is shown in fig.(i).



Fig(i) Block Diagram of the PLL system

Here,

K_D = Gain of the phase comparator or detector (unit V/rad)
F(S) = Transfer function of loop filter
A = Gain in the forward loop
K_O = Gain of the voltage controlled oscillator (unit: rad/s / V)

If a constant input voltage is applied to the VCO control input, the output frequency of the VCO remains constant. However, the phase comparator is sensitive to the difference betn the phase of the VCO output and the phase of the incoming signal. The phase of the VCO output is actually equal to the time integral of the VCO output frequency, since

$$\omega_{osc}(t) = \frac{d\phi_{osc}(t)}{dt}$$

thus,

$$\phi_{osc}(t) = \phi_{osc}\Big|_{t=0} + \int_0^t \omega_{osc}(t)\,dt$$

Thus, an integration inherently takes place within the phase locked loop. This integration is represented by the $1/s$ block as shown in fig(i).

For practical reasons, the VCO is actually designed so that when the VCO input voltage (i.e. V_o) is zero, the VCO frequency isn't zero. The relationship betn the VCO o/p frequency ω_{osc} and V_o is actually,

$$\omega_{osc} = \omega_o + K_o V_o$$

where, ω_o = free running frequency that results when $V_o = 0$.

The system as shown in fig(1) can be considered to be a classical linear feedback control system. The closed loop transfer function is given by,

$$\frac{V_0}{\phi_i} = \frac{K_D F(S) A}{1 + K_D F(S) A \cdot \frac{K_0}{S}} \qquad \left[\because H(S) = \frac{G(S)}{1+G(S)H(S)} \right]$$

$$= \frac{S K_D F(S) A}{S + K_D F(S) A \cdot K_0}$$

$$= \frac{S K_D F(S) A}{S + K_D K_0 A F(S)}$$

Usually we are interested in the response of this loop to frequency variations at the input, so that the input variable is frequency rather than phase. Since,

$$\omega_i = \frac{d\phi_i}{dt}$$

then, $\omega_i(S) = S\phi_i(S)$

and, $$\frac{V_0}{\omega_i} = \frac{1}{S} \cdot \frac{V_0}{\phi_i} = \frac{1}{S}\left[\frac{S K_D F(S) A}{S + K_D K_0 A F(S)} \right]$$

$$\therefore \frac{V_0}{\omega_i} = \frac{K_D F(S) A}{S + K_D K_0 A F(S)}$$

We consider the case in which the loop filter is removed entirely, and F(S) is unity. This is called a first-order loop and we have

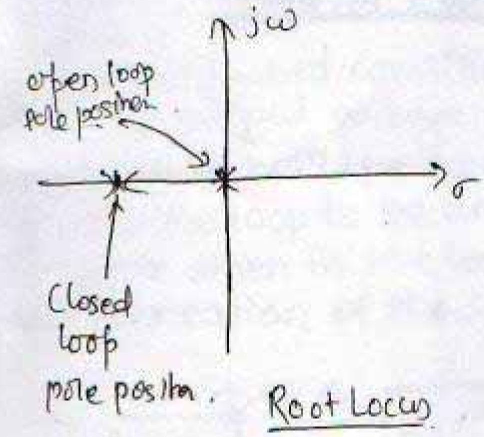
$$\boxed{\frac{V_0}{\omega_i} = \left(\frac{K_V}{S + K_V}\right)\left(\frac{1}{K_0}\right)} \quad \text{——— (A)}$$

Where, $K_V = K_0 K_D A$

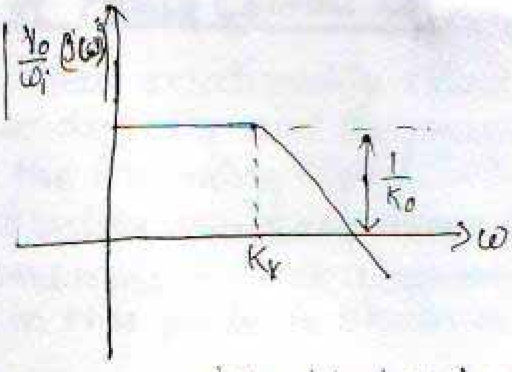
Thus, the loop inherently produces a first order, low pass transfer characteristics.

We regard the input variable as the frequency ω_i of the incoming signal. The response calculated in eq^n(A) is the response from the frequency modulation on the incoming carrier to the loop output voltage.

The constant $K_V = K_0 K_D A$ is termed as the loop bandwidth.



Root Locus



closed loop response

fig:- Root Locus and frequency response of first order PLL

Note:-

→ Operating the PLL with no loop filter has several practical drawbacks. Since, the phase detector is a multiplier, it produces a sum frequency component at its output as well as difference frequency component. The component at twice the carrier frequency will be fed directly to the output if there is no loop filter. Also, all the out of band interfering signals present at the input will appear shifted in frequency, at the output. Thus a loop filter is very desirable in applications where interfering signals are present.