13. Amplitude Modulation:

Amplitude modulation is defined as the process in which the amplitude of the carrier wave is varied with the message signal m(t). The message signal *m(t)* is an electrical signal that may represent either an audio signal, or a still image, or a video signal. Such a signal is assumed to be a lowpass signal with frequency content that extends from $f = 0$ to some upper frequency limit, say, B Hz. Hence, if the voltage spectrum (Fourier transform) *m(t)* is denoted as $M(f)$, then $M(f) = 0$ for $|f| > B$. The bandwidth B of the message signal depends on the type of analog signal. For example, the bandwidth of an audio signal is typically approximately 4 kHz and that of an analog video signal is approximately 6 MHz. The carrier signal is given as,

$$c(t) = A_c \cos 2\pi f_c t$$

where, A_c is the (unmodulated) carrier amplitude and f_c is the carrier frequency. Basically, the modulation of the carrier c(t) by the message signal m(t) converts the message signal from lowpass to bandpass, in the neighborhood of the carrier f_c. This frequency translation resulting from the modulation process is performed in order to achieve one or both of the following objectives:

(1) to translate lowpass signal in frequency to the passband of the channel so that the spectrum of the frequency-translated message signal matches the passband characteristics of the channel and
(2) to accommodate for the simultaneous transmission of signals from several message sources, where each message signal modulates a different carrier and, thus, occupies a different frequency band, as in frequency division multiplexing.

13.1 Generation of AM Signal

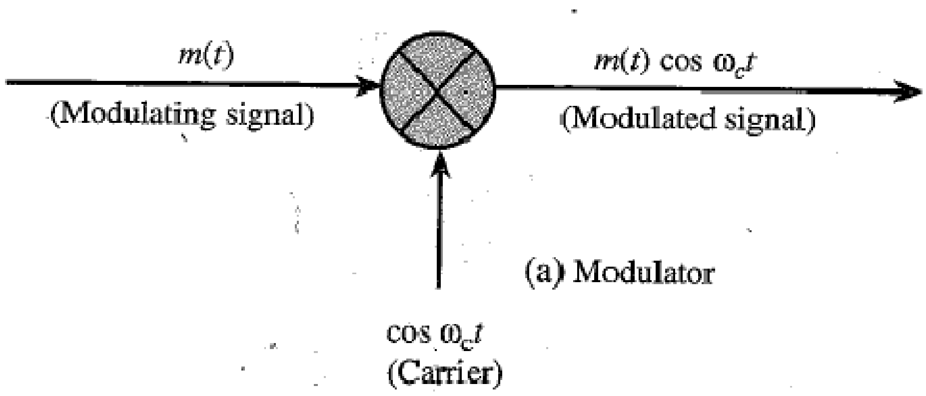


Fig.4 Generation of AM Signal

13.2 Types of AM signal

There are several different ways to modulate the amplitude of the carrier by the message signal *m(t)*, each of which results in different spectral characteristics for the transmitted signal. Specifically, these methods are called (1) *double sideband*, *suppressed-carrier AM*, (2) *conventional double sideband AM*, (3) *single sideband AM*, and (4) *vestigial sideband AM*.

13.3 Double Sideband, Suppressed-Carrier AM:

A double-sideband, suppressed-carrier (DSB-SC) AM signal is obtained by multiplying the message signal m(t) with the carrier signal c(t). Thus, we have the amplitude modulated signal

$$u(t) = m(t)c(t)= A_c m(t) \cos 2\pi f_c t$$

The voltage spectrum of the modulated signal can be obtained by computing the Fourier transform of u(t). The result of this computation is

$$U(f) = Ac/2\,[M(f - f_c) + M(f + f_c)]$$

Figure 5 illustrates the magnitude and phase spectra for M(f) and U(f). We observe that the magnitude of the spectrum of the message signal m(t) has been translated or shifted in frequency by an amount f_c. The phase of the message signal has been translated in frequency the same amount. Furthermore, the bandwidth occupancy of the amplitude modulated signal is 2B, whereas the bandwidth of the message signal m(t) is B. Therefore, the channel bandwidth required to transmit the modulated signal u(t) is $B_c = 2B$.

The frequency content of the modulated signal u(t) in the frequency band $|f| > fc$ is called the upper sideband of U(f), and the frequency content in the frequency band $|f| < fc$ is called the lower sideband of U(f). It is important to note that either one of the sidebands of U(f) contains all the frequencies that are in M(f). Thus, the frequency content of U(f) for $f > fc$ corresponds to the frequency content of M(f) for $f > 0$, and the frequency content of U(f) for $f < -fc$ corresponds to the frequency content of M(f) for $f < 0$. Hence, the upper sideband of U(f) contains all the frequencies in M(f). A similar statement applies to the lower sideband of U(f). Therefore, the lower sideband of U(f) contains all the frequency content of the message signal M(f). Since U(f) contains both the upper and the lower sidebands, it is called a double-sideband (DSB AM) signal.

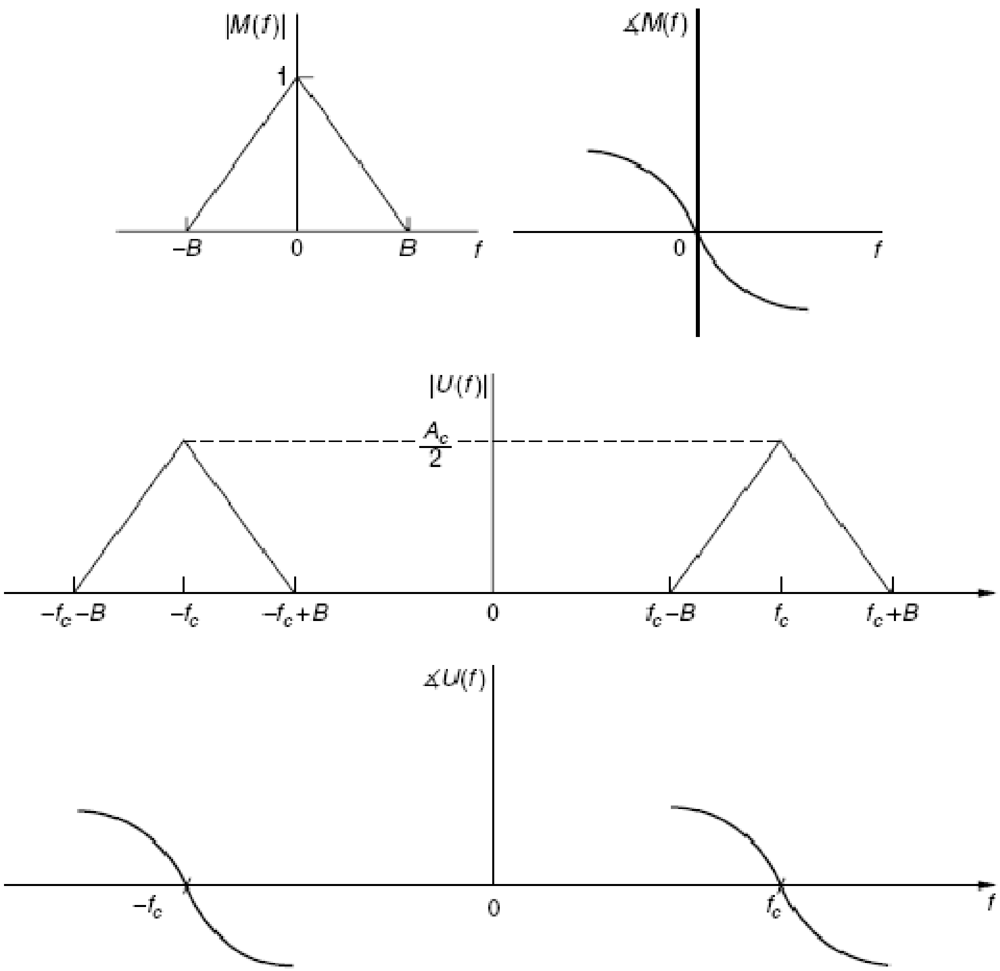


Fig.5 Magnitude and phase spectra of the message signal m(t) and the DSB AM modulated signal u(t).

The other characteristic of the modulated signal *u(t)*is that it does not contain a carrier component; that is, all the transmitted power is contained in the modulating (message) signal *m(t)*. This is evident from observing the spectrum of *U(f)*. We note that, as long as *m(t)* does not have any DC component, there is no impulse in *U(f)* at $f = f_c$, which would be the case if a carrier component was contained in the modulated signal *u(t)*. For this reason, *u(t)* is called a *suppressed-carrier signal*. Therefore, *u(t)* is a DSB-SC AM signal. In the propagation of the modulated signal through the communication channel, the signal encounters a propagation time delay, which depends on the characteristics of the propagation medium (channel). Generally, this time delay is not precisely known to the signal receiver. Such a propagation delay results in a received signal, in the absence of any channel distortion or additive noise, of the form

$$r(t) = A_c\, m(t) \cos(2\pi f_c t + \varphi_c)$$

Where, φ_c is a carrier phase manifested by the propagation delay. Suppose that we demodulate the received signal by first multiplying *r(t)* by a locally generated sinusoid cos $2\pi fct + \varphi)$, where φ is the phase of the sinusoid, and then passing the product signal through an ideal lowpass filter having a bandwidth B. The multiplication of $r(t)$ with $\cos(2\pi f c t + \varphi)$ yields

$$e(t)=r(t) \cos(2\pi fct + \varphi) = A_c\, m(t) \cos(2\pi fct + \varphi_c) \cos(2\pi fct + \varphi)$$

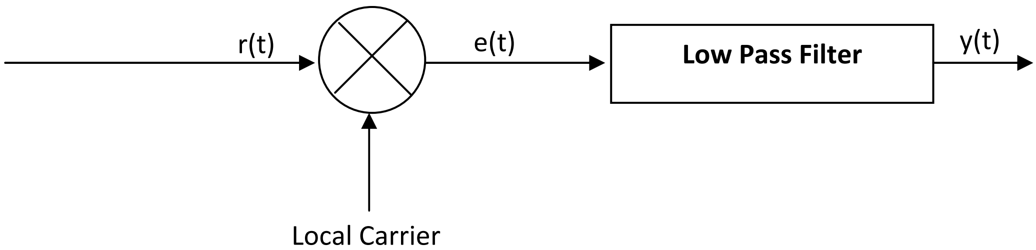
$$=A_c\, m(t) \cos(\varphi_c - \varphi) + \tfrac{1}{2} A_c\, m(t) \cos(4\pi fct + \varphi + \varphi_c)$$

A lowpass filter rejects the double frequency components and passes only the lowpass components. Hence, its output is

$$y(t) = \tfrac{1}{2} A_c\, m(t) \cos(\varphi_c - \varphi)$$

Note that $m(t)$ is multiplied by $\cos(\varphi_c - \varphi)$. Thus, the desired signal is scaled in amplitude by a factor that depends on the phase difference between the phase φ_c of the carrier in the received signal and the phase φ of the locally generated sinusoid. When φc _= φ, the amplitude of the desired signal is reduced by the factor $\cos(\varphi c - \varphi)$. If $\varphi c - \varphi = 45^\circ$, the amplitude of the desired signal is reduced by $\sqrt{2}$ and the signal power is reduced by a factor of 2. If $\varphi c - \varphi = 90^\circ$, the desired signal component vanishes.

The discussion above demonstrates the need for a *phase-coherent or synchronous demodulator* for recovering the message signal $m(t)$ from the received signal. Thus, the phase for the locally generated sinusoid should ideally be equal to the phase φ_c of the received carrier signal. Fig. below shows the block diagram of the DSB-SC demodulation process.



13.3 Double Sideband, Full-Carrier AM:

A conventional AM signal consists of a large carrier component in addition to the double-sideband AM modulated signal. The transmitted signal is expressed mathematically as

$$u(t) = A_c[1 + m(t)] \cos 2\pi f_c t$$

where, the message waveform is constrained to satisfy the condition that $|m(t)| \leq 1$. We observe that $A_c\, m(t) \cos 2\pi f_c t$ is a double-sideband AM signal and $A_c \cos 2\pi f_c t$ is the carrier component. Figure 6 illustrates an AM signal in the time domain.

As long as $|m(t)| \leq 1$, the amplitude $Ac[1 + m(t)]$ is always positive. This is the desired condition for conventional DSB AM that makes it easy to demodulate, as described next. On the other hand, if $m(t) < -1$ for some t, the AM signal is said to be over modulated and its demodulation is rendered more complex. In practice, m(t) is scaled so that its magnitude is always less than unity. The voltage spectrum of u(t) is given as,

$$U(f) = A_c/2\, [M(f - f_c) + M(f + f_c) + \delta(f - f_c) + \delta(f + f_c)]$$

This spectrum is sketched in Fig. 6. We observe that spectrum of the conventional AM signal occupies a bandwidth twice the bandwidth of the message signal. As in the case of DSB-SC carrier, conventional AM consists of both an upper sideband and a lower sideband. In addition, the spectrum of a conventional AM signal contains impulses at $f = f_c$ and $f = -f_c$, which correspond to the presence of the carrier component in the modulated signal. The major advantage of conventional AM signal transmission is the ease with which the signal can be demodulated. There is no need for a synchronous demodulator. Since the message signal m(t) satisfies the condition $|m(t)| < 1$, the envelope (amplitude) $1 + m(t) > 0$. If we rectify the received signal, we eliminate the negative values without affecting the message signal as shown in Fig. 7. The rectified signal is equal to u(t) when $u(t) > 0$ and zero when $u(t) < 0$. The message signal is recovered by passing the rectified signal through a lowpass filter whose bandwidth matches that of the message signal. The combination of the rectifier and the lowpass filter is called an envelope detector.

Ideally, the output of the envelope detector is of the form

$$d(t) = g_1 + g_2 m(t)$$

Where, g_1 represents a dc component and g_2 is a gain factor due to the signal demodulator. The dc component can be eliminated by passing d(t) through a transformer, whose output is $g_2 m(t)$. The simplicity of the demodulator has made conventional DSB AM a practical choice for AM radio broadcasting. Since there are literally billions of radio receivers, an inexpensive implementation of the demodulator is extremely important. The power inefficiency of conventional AM is justified by the fact that there are few broadcast transmitters relative to the number of receivers. Consequently, it is cost-effective to construct powerful transmitters and sacrifice power efficiency in order to simplify the signal demodulation at the receivers.

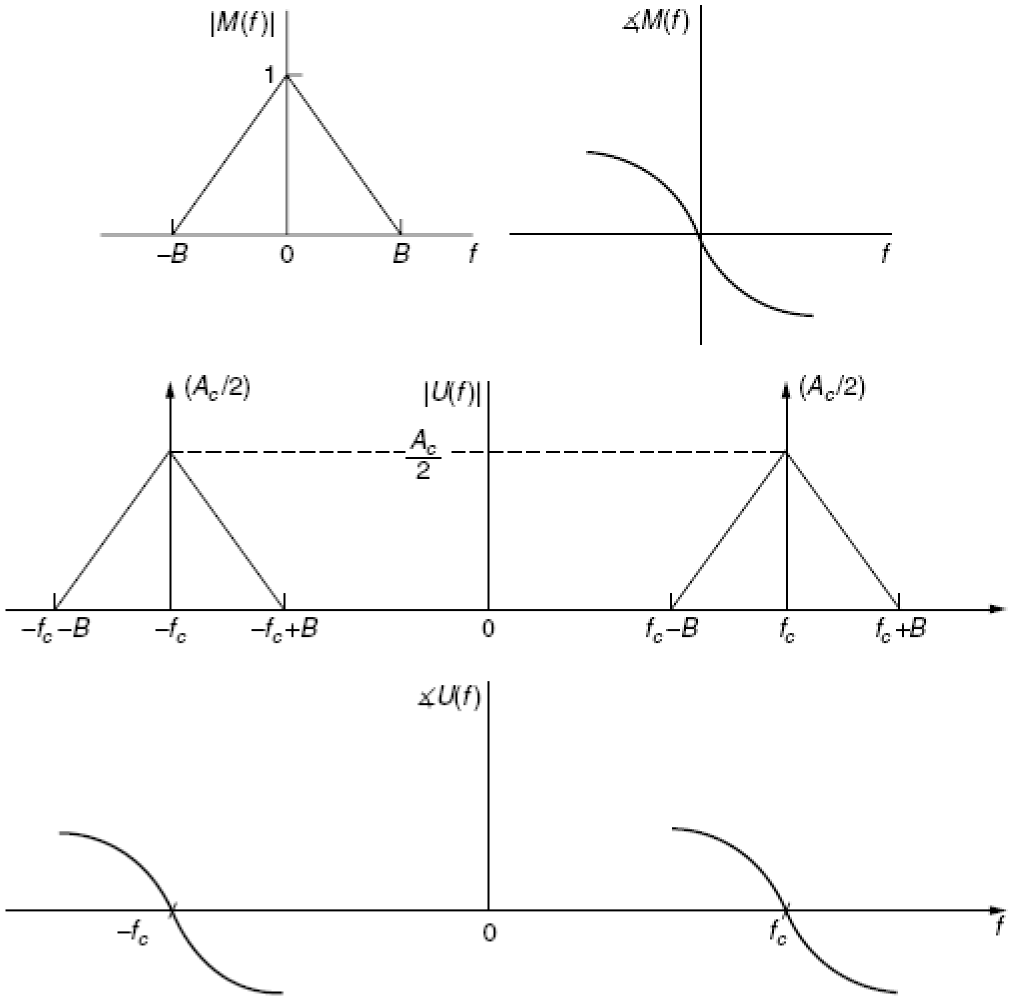


Fig.6 Magnitude and phase spectra of the message signal m(t) and the conventional AM signal.

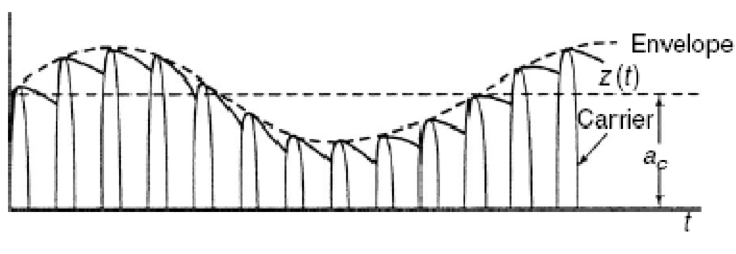


Fig.7 Envelope detection of conventional AM signal.

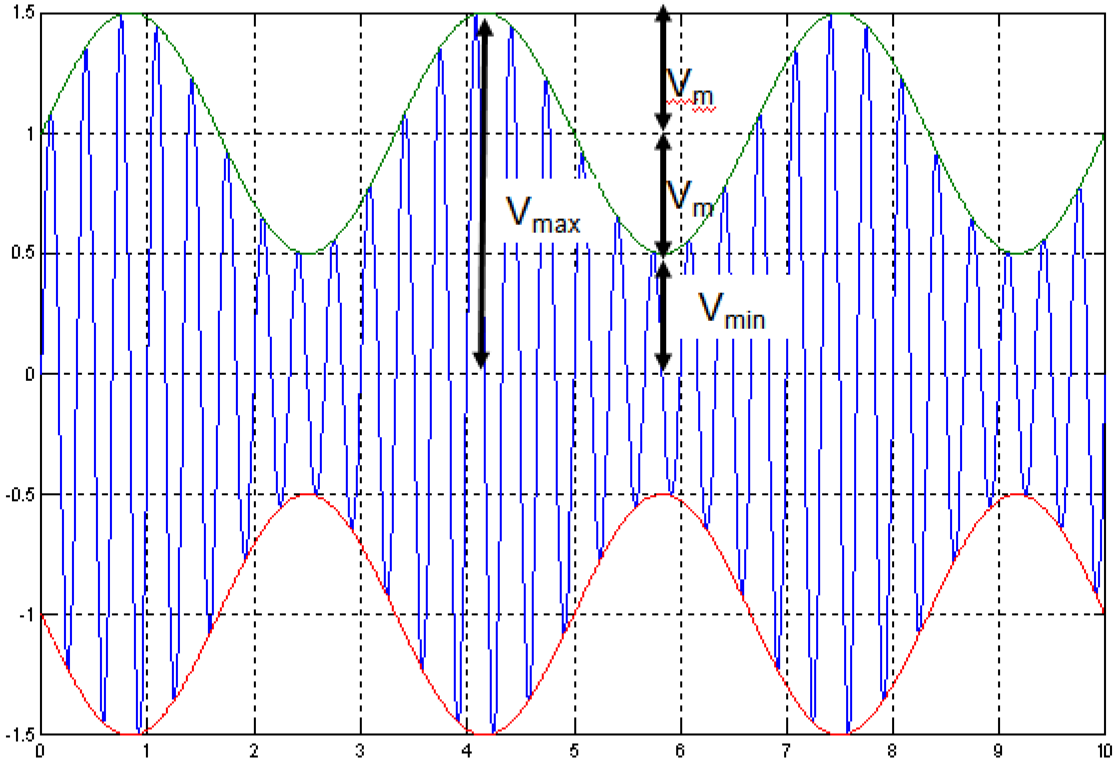
13.4 Single Sideband AM (SSB)

It was observed that a DSB-SC AM signal required a channel bandwidth of $B_c = 2B$ for transmission, where B is the bandwidth of the message signal m(t). However, the two sidebands are redundant. The transmission of either sideband is sufficient to reconstruct the message signal m(t) at the receiver. Thus, the bandwidth of the transmitted signal is reduced to that of the message signal m(t). i.e. the channel bandwidth required for SSB transmission is B.

13.5 Vestigial Sideband AM (VSB)

The stringent frequency-response requirements on the sideband filter in a SSB AM system can be relaxed by allowing a part, called a vestige, of the unwanted sideband to appear at the output of the modulator. Thus, the design of the sideband filter is simplified at the cost of a modest increase in the channel bandwidth required to transmit the signal. The resulting signal is called vestigial-sideband (VSB) AM. The video signal in TV broadcasting is transmitted through VSB AM.

14. Modulation Index in AM signal



$$V_m = mV_c \Rightarrow m = \frac{V_m}{V_c}$$

$$V_m = \frac{V_{max} - V_{min}}{2}$$

$$V_c = V_{max} - V_m$$

$$V_c = \frac{V_{max} + V_{min}}{2}$$

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

V_{USB} = Peak Amplitude of Upper side band = $V_m/2$ = ¼ (Vmax-Vmin)

V_{LSB} = Peak Amplitude of Upper side band = $V_m/2$ = ¼ (Vmax-Vmin)

Numerical Problems:

Prob1. For an AM DSB-FC modulator, with carrier frequency of 100 KHz, and a maximum modulating signal of 5 KHz, Determine:

i. Frequency limits of the upper and lower side bands
ii. Bandwidth of the modulated signal
iii. upper and lower side frequencies when modulating signal is a 3 kHz tone

Solution of i)

(100 - 5) kHz TO 100 kHz = 95 kHz TO 100 kHz = LSB

100 kHz TO (100 + 5) kHz = 100 kHz TO 105 kHz = USB

Solution of ii)

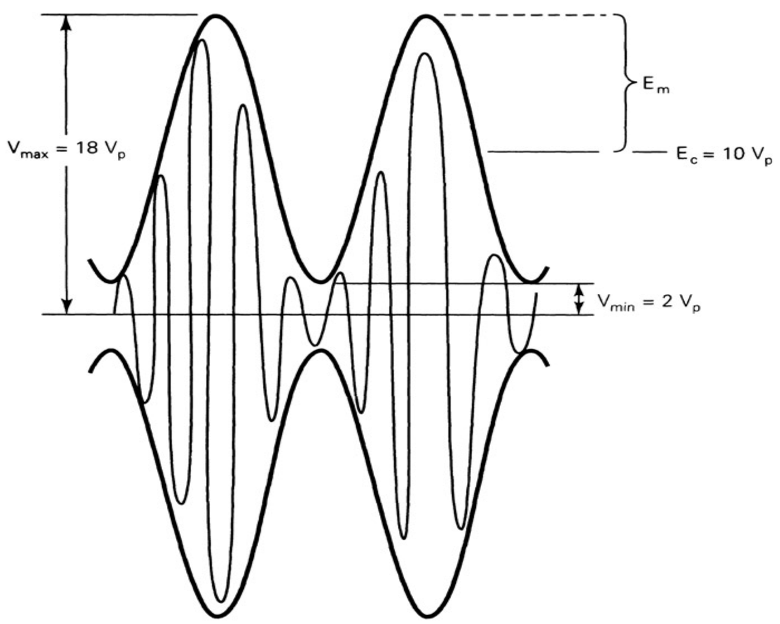
B_c = 2B = 2 x 5 kHz = 10 kHz

Solution of iii)

(100 - 3) kHz = 97 kHz = LSB

(100 + 3) kHz = 103 kHz = USB

Prob 2. Determine the following from the AM waveform



a) **peak amplitude of the upper and lower side frequencies**

b) **peak amplitude of the unmodulated carrier**
c) **peak change in the amplitude of the envelope**
d) **Modulation index**
e) **Percentage Modulation**

Solution:

a) $E_{usb} = E_{lsb} = \frac{E_m}{2} = \frac{1}{4}(V_{\max} - V_{\min})$

$E_{usb} = E_{lsb} = \frac{1}{4}(18 - 2) = 4V$

b) $E_c = \frac{1}{2}(V_{\max} + V_{\min}) = \frac{1}{2}(18 + 2) = 10V$

c) $E_m = \frac{1}{2}(V_{\max} - V_{\min}) = \frac{1}{2}(18 - 2) = 8V$

d) $m = \frac{E_m}{E_c} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} = \frac{8}{10} = 0.8$

e) $M = \frac{E_m}{E_c} x\,100\% = 0.8x100\% = 80\%$

Or

$$M = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} x100\% = \frac{18 - 2}{18 + 2} x100\% = 80\%$$

15. Power in an Amplitude Modulation

The total power in an AM is given as,

$$P_t = \frac{V_{carr}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

Power of carrier is,

$$P_c = \frac{V_{carr}^2}{R} = \frac{\left(V_c/\sqrt{2}\right)^2}{R} = \frac{V_c^2}{2R}$$

$$Vusf = Vlsf = \frac{Vm}{2} = \frac{mVc}{2}$$

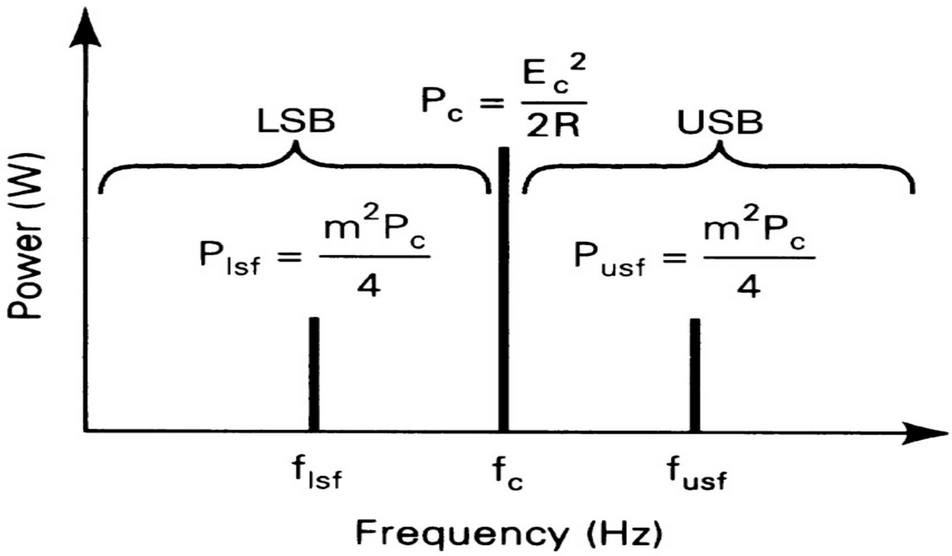
$$P_{usb} = P_{lsb} = \frac{m^2 V_c^2}{8R} = \frac{m^2}{4} P_c$$

$$P_t = P_c + P_{usb} + P_{lsb}$$

$$P_t = P_c + \frac{m^2 P_c}{4} + \frac{m^2 P_c}{4} = P_c + \frac{m^2 P_c}{2}$$

$$P_t = P_c(1+\frac{m^2}{2})$$

Power of the carrier is unaffected by the modulation process. $\uparrow \beta \uparrow P_t$



WITH 100% MODULATION,

$$P_t = P_c(1+\frac{1}{2}) = 1.5P_c$$

$$P_{usb} + P_{lsb} = \frac{1}{2}P_c$$

DSB-FC disadvantage: the information is contained in the sidebands although most of the power is wasted in the carrier (DSB-SC eliminates this disadvantage).

The advantage of envelope detection in am has its price. In am, the carrier component does not carry any information; hence, the carrier power is wasted.

Prob. 3

For an AM DSB-FC wave with a peak un-modulated carrier voltage Vc = 10 vp, a load resistance of R_L= 10 ohms, and a modulation index of 1, determine:

a) **Carrier Power**
b) **Upper and Lower Sideband Power**
c) **Total sideband Power**
d) **Total Power in the modulated wave**
e) **Draw the power spectrum**

Solution

a. $P_c = \frac{V_c^2}{2R} = \frac{10^2}{2(10)} = 5W$

b. $Pusb = Plsb = \frac{m^2}{4} Pc = \frac{1(5)}{4} = 1.25W$

c. $P_{usb} + P_{lsb} = \frac{\beta^2}{2} P_c = \frac{1(5)}{2} = 2.5\,W$

d. $P_t = P_c(1 + \frac{m^2}{2}) = 5(1 + \frac{1^2}{2}) = 7.5W$